A STATISTICAL PERFORMANCE INDICATOR IN SOME IMAGE PROCESSING PROBLEMS

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ABSTRACT

The ability to compare or relate two digital images may be useful in developing performance evaluation algorithms. This thesis investigates the use of a particular correlation measure, R_p^2 developed from the multidimensional unreplicated linear functional relationship (MULFR) model with single slope, as a measure or indicator of performance. This MULFR model is an extended version of the ULFR model introduced by Adcock in 1877. A literature survey was carried out showing that R_n^2 has not been used before. The coefficient R_p^2 was investigated in its ability to handle the issues of non-perfect reference image, multiple image attributes and combining image local-global information simultaneously. This survey is followed with the maximum likelihood estimation of parameters and a brief discussion of some theoretical properties of R_p^2 . To investigate robust properties of R_p^2 , an extensive simulation exercise was then carried out. Promising results, thus far, motivate the use of R_p^2 in two image analysis problems; firstly a character recognition problem and secondly a particular data compression problem. In a handwritten Chinese character recognition problem, the R_n^2 achieved the highest recognition rates even the pre-processing stage is removed from the recognition system. A substantial reduction of processing time, approximately 40.36% to 75.31%, was achieved using R_p^2 . In JPEG compression problem, R_p^2 is used as a measure of image quality which in turn indicates the performance of the compression method. It is shown that R_p^2 performs well and satisfies the monotonicity, accuracy and consistency properties when perfect reference image was used. R_p^2 was also shown to perform better than some frequently used similarity measures when imperfect reference image was used.

ABSTRAK

Keupayaan membanding dua keping imej berdigital merupakan satu unsur penting dalam rekaan algoritma bagi tujuan penilaian pretasi pemprosesan imej. Thesis ini mengkaji penggunaan ukuran korelasi tertentu, R_p^2 , yang diperolehi daripada model hubungan fungsian linear multidimensi tak replika (MULFR) dengan satu kecendurungan sebagai satu ukuran atau penunjuk prestasi. Model MULFR ini merupakan satu lanjutan dari model ULFR yang diperkenalkan oleh Adcock pada tahun 1877. Satu tinjauan literasi yang dijalankan mendapati bahawa R_p^2 belum pernah digunakan sebelum ini. Keupayaan pekali R_p^2 menangani tiga isu utama serentak juga disiasat. Isu-isu ini ialah isu imej rujukan yang tidak sempurna, isu kepelbagaian sifat imej dan isu gabungan maklumat imej setempat-sejagat. Tinjauan tersebut disusuli dengan anggaran parameter-parameter dengan kaedah penganggaran kebolehjadian maksimum dan perbincangan ringkas terhadap ciri-ciri R_p^2 secara teori. Satu simulasi yang ekstensif juga dijalankan bagi mengkaji ciri-ciri keteguhan R_p^2 . Hasil simulasi yang memuaskan telah mengalak penggunaan R_p^2 dalam dua bidang imej analysis; bidang pertama ialah masalah pengenalpastian huruf dan bidang kedua ialah masalah mampatan data. Dalam masalah pengenalpastian tulisan huruf Cina, pekali R_p^2 mencapai kadar pengenalpastian yang paling tinggi walaupun peringkat prapemprosesan dikeluarkan dari system pengenalpastian tersebut. Satu pengurangan masa pemprosesan yang banyak, kira-kira 40.36% hingga 75.31% telah dicapai dengan menggunakan R_p^2 . Dalam masalah mampatan JPEG, R_p^2 digunakan sebagai satu ukuran kualiti imej yang selanjutnya dapat menunjukkan prestasi kaedah mampatan tersebut. Didapati bahawa R_p^2 mempunyai prestasi yang baik dan memenuhi cirri-ciri ekanada, ketepatan dan konsisten apabila imej rujukan yang sempurna digunakan. Didapati R_n^2 juga mempunyai prestasi yang lebih baik berbanding dengan beberapa sukatan keserupaan yang kerap digunakan apabila imej rujukan yang tidak sempurna dipakai.

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List of Frequently Used Abbreviations

CBDD	City block distance with deviation
CMF	Compound mahalanobis function
CHI2	Chi-Square measure
COD	Coefficient of determination
FR	Full reference
HCCR	Handwritten Chinese character recognition
ISM	Image similarity measure
JPEG	Joint photographic experts group
MD	Minimum distance
ML	Maximum likelihood
MOS	Mean opinion score
MQDF	Modified quadratic discriminant function
MSSIM	Mean structural similarity
MULFR	Multidimensional unreplicated linear functional relationship
NR	No reference
PSNR	Peak signal to noise ratio
Rf2/ R_F^2	Coefficient of determination for unreplicated linear functional relationship model
RMSE	Root mean square error
$\operatorname{Rp2}/R_p^2$	Coefficient of determination for multidimensional unreplicated linear functional relationship model
RR	Reduced reference
$\operatorname{Rs2}/R_S^2$	Coefficient of determination for simple linear regression model
SISM	Statistical image similarity measure
ULFR	Unreplicated linear functional relationship

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