A STATISTICAL PERFORMANCE INDICATOR IN SOME IMAGE PROCESSING PROBLEMS

CHANG YUN FAH

THESIS SUBMITTED IN FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

INSTITUTE OF MATHEMATICAL SCIENCES
FACULTY OF SCIENCE
UNIVERSITY OF MALAYA
KUALA LUMPUR

2012
ABSTRACT

The ability to compare or relate two digital images may be useful in developing performance evaluation algorithms. This thesis investigates the use of a particular correlation measure, $R_p^2$ developed from the multidimensional unreplicated linear functional relationship (MULFR) model with single slope, as a measure or indicator of performance. This MULFR model is an extended version of the ULFR model introduced by Adcock in 1877. A literature survey was carried out showing that $R_p^2$ has not been used before. The coefficient $R_p^2$ was investigated in its ability to handle the issues of non-perfect reference image, multiple image attributes and combining image local-global information simultaneously. This survey is followed with the maximum likelihood estimation of parameters and a brief discussion of some theoretical properties of $R_p^2$. To investigate robust properties of $R_p^2$, an extensive simulation exercise was then carried out. Promising results, thus far, motivate the use of $R_p^2$ in two image analysis problems; firstly a character recognition problem and secondly a particular data compression problem. In a handwritten Chinese character recognition problem, the $R_p^2$ achieved the highest recognition rates even the pre-processing stage is removed from the recognition system. A substantial reduction of processing time, approximately 40.36% to 75.31%, was achieved using $R_p^2$. In JPEG compression problem, $R_p^2$ is used as a measure of image quality which in turn indicates the performance of the compression method. It is shown that $R_p^2$ performs well and satisfies the monotonicity, accuracy and consistency properties when perfect reference image was used. $R_p^2$ was also shown to perform better than some frequently used similarity measures when imperfect reference image was used.
ABSTRAK

ACKNOWLEDGEMENTS

Firstly, I would like to express my deepest appreciation and gratitude to my supervisor, Associate Professor Dr. Omar Mohd Rijal and my co-supervisor, Associate Professor Dr. Syed Abdul Rahman Syed Abu Bakar. They have given me invaluable advice, supervision and coaching all these years.

Secondly, I would like to express my deepest gratitude and love to my beloved wife, Ming See and my parents who have continuously support and encourage me in this time-consuming and difficult work. Without them, I may take longer time to complete my research. Not forgetting my two little girls, Zhi Shin and Zhi Xin, their smiles have energized me to keep on working after a tire day.

Last but not least, my sincere thanks to all the members of the Institute of Mathematical Sciences, University of Malaya and Department of Mathematical Sciences, Universiti Tunku Abdul Rahman. In particular, I would like to thank Dr. Loo Tee Haw, Associate Professor Norliza bt. Mohd Noor and Dr. Liew How Hooi for their useful advice and support.
## CONTENTS

Abstract

Abstrak

Acknowledgements

Contents

List of Frequently Used Abbreviations

List of Tables

List of Figures

### Chapter 1 Introduction

1.1 Type of Images

1.2 Image Processing Procedures

1.3 Problem Related to Comparing Two Images

1.3.1 No Universal Similarity Measure for All Applications

1.3.2 Conditions of Using Similarity Measure

1.3.3 Level of Difficulty in Comparing Images

1.4 Objectives of the Study

1.5 Thesis Structure and Organization

### Chapter 2 Literature Survey on Statistical Image Similarity Measures for Comparing Two Images

2.1 Introduction

2.2 Other Surveys Done

2.3 Chronological Survey of SISMs and Their Applications

2.3.1 Summary Comments on FR-SISMs

2.4 Issues Related to FR-SISM

2.4.1 Issue One: The Need to Consider Full Reference Image that Subject to Error

2.4.2 Issue Two: The Need to Compare Images Using Multiple Image Attributes

2.4.3 Issue Three: The Need to Combine Local and Global Image Information

2.5 Properties of the Selected SISMs and Their Strengths and Limitations

2.5.1 Properties of the Selected SISMs

2.5.2 Summary of the Selected SISMs

### Chapter 3 Unreplicated Linear Functional Relationship Model

3.1 Why Some Regression Models Are Not Suitable?

3.2 Linear Functional Relationship Models

3.2.1 Basic Definition of Unreplicated Linear Functional Relationship Model

3.2.2 A Brief Historical Remarks
Chapter 4  Multidimensional Unreplicated Linear Functional Relationship Model with Single Slope and its Coefficient of Determination 71

4.1  Formulation of Multidimensional ULFR (MULFR) with Single Slope
  4.1.1  The MULFR Model 72
  4.1.2  Estimation of Parameters 74
  4.1.3  Graphical Representation of the MULFR Model with Single Slope 80

4.2  Properties of Parameters 82
  4.2.1  Unbiasedness of Parameters 82
  4.2.2  Variance and Covariance of the Expected Parameters 85
  4.2.3  Consistent Estimators 87
  4.2.4  Asymptotic Normality and Efficiency 88
  4.2.5  Interval Estimation for \( \alpha \) and \( \beta \) 91

4.3  Coefficient of Determination for MULFR Model 92

4.4  Properties of Coefficient of Determination when \( \lambda = 1 \)
  4.4.1  Range: \( 0 \leq R_p^2 \leq 1 \) (Boundedness and Nonnegative) 94
  4.4.2  Range of \( R_p^2 \) (an improvement of Section 4.4.1) 94
  4.4.3  Non-Symmetric Property 95
  4.4.4  Identity of Indiscernible (Self-Distance) 97
  4.4.5  Translation Invariant 97
  4.4.6  Scale Invariant 98
  4.4.7  Confidence Interval 99
  4.4.8  \( R_p^2 \) is a Special Case of \( R_p^2 \) When \( p = 1 \) 100

4.5  Conclusion 101

Chapter 5  Simulation Study 103

5.1  Selection of Parameters Values 103
  5.1.1  Selection of \( \alpha = [\alpha_1, \ldots, \alpha_p] \) 103
  5.1.2  Selection of \( \beta \) 104
  5.1.3  Selection of \( p \) and \( n \) 104
  5.1.4  Selection of \( \sigma \) and \( \lambda \) 105
  5.1.5  Selection of Distributions for \( \epsilon_i \) and \( \delta_i \) 105

5.2  Monte Carlo Simulation 106
  5.2.1  Monte Carlo Simulation Procedure 106
  5.2.2  Transformation to a Non-Normal Distribution 108

5.3  Simulation A: Performance of the MULFR Model When the Basic Assumptions are Satisfied 110
5.3.1 Both $\delta$ and $\varepsilon$ are Normally Distributed, $\lambda = 1$
and $\sigma = 1$, $\beta = 1$ 111
5.3.2 Repeating Section 5.3.1 to Compare
$\beta = 1.5, 10, 40$ 112
5.3.3 Both $\delta$ and $\varepsilon$ are Normally Distributed, $\lambda = 1$,
$\beta = 10$ and $\sigma = 1, 5, 10$ 115
5.4 Simulation B: Robustness of the MULFR Model When
the Basic Assumptions are Violated 117
5.4.1 Robustness of $\hat{\beta}$ and $R^2_p$ to Non-Normality 118
5.4.2 Robustness of $\hat{\beta}$ and $R^2_p$ to $\lambda \neq 1$ When $\beta = 10$
and $\sigma = 1$ 122
5.5 Properties of Maximum Likelihood Estimator, $\hat{\beta}$ 126
5.6 Empirical Distribution of $R^2_p$ 127
5.7 Summary 134

Chapter 6 Online Handwritten Chinese Character Recognition 136
6.1 Handwritten Chinese Character Recognition 136
6.2 Database 138
6.3 The Experiment 140
6.3.1 Cropping 142
6.3.2 Normalization 142
6.3.3 $X$-Graph and $Y$-Graph 143
6.3.4 Properties of the $X$-Graph and $Y$-Graph 145
6.4 Haar Wavelet Transform 146
6.5 Classification 147
6.5.1 Rough Classification 147
6.5.2 Fine Classification 148
6.6 Results and Discussions 151
6.6.1 Recognition Rate (Accuracy) and Precision 151
6.6.2 Processing Time 153
6.6.3 Feature Size and Storage Space 157
6.7 Verification of the Experimental Results 158
6.8 Summary 160

Chapter 7 Image Quality Assessment for JPEG Compression 162
7.1 Test Images 164
7.2 Type of Image Distortions 165
7.3 Selection of Image Quality Attributes 167
7.4 Performance of $R^2_p$ and $R^2_p$ When Reference Image
Has Perfect Quality 169
7.4.1 Properties for a Good Image Similarity Measure 169
7.4.2 Experiments 172
7.5 Performance of $R^2_p$ and $R^2_p$ When Reference Image Has
Imperfect Quality 176
7.6 Estimation of Percentage of Distorted Pixels Using $R_F^2$ and $R_P^2$

7.7 Summary

Chapter 8 Concluding Remarks

8.1 Research Conclusion
8.1.1 Findings of Literature Review
8.1.2 Theoretical Properties of $R_P^2$
8.1.3 Simulation to Verify Sampling Properties of $\hat{\alpha}$, $\hat{\beta}$ and $R_P^2$
8.1.4 Application in Character Recognition
8.1.5 Application in Image Compression

8.2 Areas of Further Research
8.2.1 Study the MULFR Model with Correlated Errors
8.2.2 Different Regression Methods
8.2.3 Exploring More Applications of $R_P^2$ in Image Processing

8.3 Published Articles

References

Appendix A1: Simulation Results for Both $\delta$ and $\varepsilon$ are Normally Distributed, $\lambda = 1$, $\sigma = 1$ and $\beta = 10, 40$

Appendix A2: Simulation Results for Both $\delta$ and $\varepsilon$ are Normally Distributed, $\lambda = 1$, $\beta = 10$ and $\sigma = 5, 10$

Appendix A3: Robustness of $\hat{\beta}$ and $R_P^2$ To Non-Normality

Appendix A4: Robustness of $\hat{\beta}$ and $R_P^2$ To $\lambda \neq 1$ When $\beta = 10$ and $\sigma = 1$

Appendix B1: JPEG Codec Images with Compression Factor 10 to 100 for Four Selected Image
# List of Frequently Used Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBDD</td>
<td>City block distance with deviation</td>
</tr>
<tr>
<td>CMF</td>
<td>Compound mahalanobis function</td>
</tr>
<tr>
<td>CHI2</td>
<td>Chi-Square measure</td>
</tr>
<tr>
<td>COD</td>
<td>Coefficient of determination</td>
</tr>
<tr>
<td>FR</td>
<td>Full reference</td>
</tr>
<tr>
<td>HCCR</td>
<td>Handwritten Chinese character recognition</td>
</tr>
<tr>
<td>ISM</td>
<td>Image similarity measure</td>
</tr>
<tr>
<td>JPEG</td>
<td>Joint photographic experts group</td>
</tr>
<tr>
<td>MD</td>
<td>Minimum distance</td>
</tr>
<tr>
<td>ML</td>
<td>Maximum likelihood</td>
</tr>
<tr>
<td>MOS</td>
<td>Mean opinion score</td>
</tr>
<tr>
<td>MQDF</td>
<td>Modified quadratic discriminant function</td>
</tr>
<tr>
<td>MSSIM</td>
<td>Mean structural similarity</td>
</tr>
<tr>
<td>MULFR</td>
<td>Multidimensional unreplicated linear functional relationship</td>
</tr>
<tr>
<td>NR</td>
<td>No reference</td>
</tr>
<tr>
<td>PSNR</td>
<td>Peak signal to noise ratio</td>
</tr>
<tr>
<td>$R^2_F$</td>
<td>Coefficient of determination for unreplicated linear functional relationship model</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root mean square error</td>
</tr>
<tr>
<td>$R^2_p$</td>
<td>Coefficient of determination for multidimensional unreplicated linear functional relationship model</td>
</tr>
<tr>
<td>RR</td>
<td>Reduced reference</td>
</tr>
<tr>
<td>$R^2_S$</td>
<td>Coefficient of determination for simple linear regression model</td>
</tr>
<tr>
<td>SISM</td>
<td>Statistical image similarity measure</td>
</tr>
<tr>
<td>ULFR</td>
<td>Unreplicated linear functional relationship</td>
</tr>
</tbody>
</table>
List of Tables

Table 2.1: Full Reference SISM from year 1980 to 2010. 21
Table 2.2: No Reference and Reduced Reference SISM. 25
Table 2.3: Comparison of the number of Statistical based and Non-Statistical based ISMs for different applications from year 1980 to 2010. 26
Table 2.4: Properties of the selected SISMs. Y=yes, N=no. S=single attribute is used, B=bivariate attributes are used, M=multiple attributes are used, L=local measure, G=global measure. 51
Table 3.1: Comparing different correlation based metrics. 56
Table 5.1: All model assumptions satisfied with $\lambda = 1, \sigma = 1, \beta = 1$. 111
Table 5.2: All model assumptions satisfied with $\lambda = 1, \sigma = 1, \beta = 1.5$. 112
Table 5.3: Parameters estimation, standard deviation, mean square error and length of confidence interval for $\hat{\beta}$ and $R_p^2$ involving $\lambda = 1, \sigma = 1, \beta = 1, p = 1, n = 10$ and varying distributions. 118
Table 5.4: Summary for Kolmogorov-Smirnov test. 134
Table 6.1: Three commonly used databases for Chinese character recognition and a new created database for this research. 139
Table 6.2: Experimental results for different writers: (a) with pre-processing and (b) without pre-processing. Each writer writes all 3000 different Chinese characters. 152
Table 6.3: Experimental results for different number of strokes: (a) writer A and (b) writer B. Number of strokes is grouped into three categories: less than 6 strokes, between 6 to 12 strokes, and more than 12 strokes. 153
Table 6.4: Result of processing time by components. 154
Table 6.5: Processing steps for four different feature extraction methods. 155
Table 6.6: Example of processing time with and without rough classifications. 155
Table 6.7: Distribution of the number of strokes. 157
Table 6.8: Reduced time rates in comparing the algorithm of $R_p^2$ with CBDD, MD, MQDF and CMF, $R_p^2$, $R_2^2$, MSSIM and RMSE. 157
Table 6.9: Feature sizes for four different feature extraction methods. 158
Table 6.10: Recognition rates without normalization based on HCH-GB1 dataset. 159
Table 7.1: Information on the frequently used test image. 164
Table 7.2: Average Spearman correlation between similarity values and compression factors. 173
List of Figures

Figure 1.1: Matrix representation of an $M \times N$ image. 3
Figure 1.2: Schematic formation of RGB colour image. 3
Figure 1.3: Schematic formation of indexed colour image. 4
Figure 1.4: Temporal sequence of intensity values for a digital video. 5
Figure 2.1: Summary of the FR, RR and NR-SISMs from year 1980 – 2010 for different original applications. 26
Figure 2.2: Number of FR-SISMs for various statistical approaches from year 1980 to 2010. 27
Figure 2.3: Non-perfect Lena reference image. (a) with Gaussian noise $N(0,0.001)$, (b) with Gaussian noise $N(0,0.01)$, and (c) with Gaussian noise $N(0,0.05)$. 29
Figure 2.4(a): Image similarity values obtained from Fig. 2.3(a). 30
Figure 2.4(b): Image similarity values obtained from Fig. 2.3(b). 30
Figure 2.4(c): Image similarity values obtained from Fig. 2.3(c). 30
Figure 2.5(a): $R^2_S$ measured of mean and variance for the compressed Lena image. 31
Figure 2.5(b): Chi-square measured of mean and variance for the compressed Lena image. 32
Figure 2.6: Original Lena image (Left). Distorted Lena image with Gaussian noise $N(0,0.001)$ (Middle). Distorted Lena image with Gaussian noise $(0,0.001)$ and extreme value. 33
Figure 3.1: Simulation values for $R^2_S$ and $R^2_F$ with (a) $n = 10$, (b) $n = 50$, (c) $n = 100$, (d) $n = 1000$, (e) $n = 5000$, (f) $n = 10000$. 68
Figure 4.1: Figure 4.1: Graphical representation of MULFR model. The parallel bolded lines in red color are the fitted model with the same slope with different intercept values. 81
Figure 5.1: Probability density function of $\varepsilon$ with $(\bar{m}_x, \bar{m}_y) = (0.0, 3.0)$, $(0.0, 12.0)$ and $(1.5, 6.7)$. 109
Figure 5.2: Probability density function of $\varepsilon$ with $(\bar{m}_x, \bar{m}_y) = (0.0, 3.0)$, $(1.5, 6.7)$, $(3.0, 16.0)$. 110
Figure 5.3: Probability density function of $\varepsilon$ with $(\bar{m}_x, \bar{m}_y) = (0.0, 3.0)$, $(-1.5, 7.5)$, $(-3.0, 16.7)$. 110
Figure 5.4: Mean square error for $\hat{\beta}$ when $\lambda = 1, \sigma = 1$ and $\beta = 1.5, 10, 40$. 113
Figure 5.5: Length of confidence interval for $R^2_p$ when $\lambda = 1, \sigma = 1$ and $\beta = 1.5, 10, 40$. 114
Figure 5.6: Mean square error for $\hat{\beta}$ when $\lambda = 1, \beta = 10, \sigma = 1, 5, 10$. 116
Figure 5.7:  Length of confidence interval for $R_p^2$ when $\lambda = 1, \beta = 10$, $\sigma = 1, 1.5, 10$.

Figure 5.8:  Mean square error for $\hat{\beta}$ under varying error distributions when $p = 1, 2, 5$ and $n = 10, 100, 1000$. (a) Normal distribution and fourteen non-normal distributions with varying positive skewness and kurtosis values. (b) Twelve non-normal distributions with varying negative skewness and kurtosis values.

Figure 5.9:  Length of confidence interval for $R_p^2$ under varying error distributions when $p = 1, 2, 5$ and $n = 10, 100, 1000$. (a) Normal distribution and fourteen non-normal distributions with varying positive skewness and kurtosis values. (b) Twelve non-normal distributions with varying negative skewness and kurtosis values.

Figure 5.10:  Mean square error for $\hat{\beta}$ when $\sigma = 1, \beta = 10, \lambda = 1.0, 1.5, 10, 30, 100$.

Figure 5.11:  Length of confidence interval for $R_p^2$ when $\sigma = 1, \beta = 10$, $\lambda = 1.0, 1.5, 10, 30, 100$.

Figure 5.12:  Consistency of $\hat{\beta}$.

Figure 5.13:  Empirical cumulative density functions of $R_p^2$ at high correlation value (average $R_p^2 \approx 0.99$) and CDF for standard normal distribution. Given $p = 1$.

Figure 5.14:  Empirical cumulative density functions of $R_p^2$ at high correlation value (average $R_p^2 \approx 0.99$) and CDF for standard normal distribution. Given $p = 5$.

Figure 5.15:  Empirical cumulative density functions of $R_p^2$ at moderate correlation value (average $R_p^2 \approx 0.56$) and CDF for standard normal distribution. Given $p = 1$.

Figure 5.16:  Empirical cumulative density functions of $R_p^2$ at moderate correlation value (average $R_p^2 \approx 0.58$) and CDF for standard normal distribution. Given $p = 5$.

Figure 5.17:  Empirical cumulative density functions of $R_p^2$ at low correlation value (average $R_p^2 \approx 0.1$) and CDF for standard normal distribution. Given $p = 1$.

Figure 5.18:  Empirical cumulative density functions of $R_p^2$ at low correlation value (average $R_p^2 \approx 0.1$) and CDF for standard normal distribution. Given $p = 5$.

Figure 6.1:  Examples of 50 normalized Chinese characters in songti written style.

Figure 6.2:  Wacom Intuos® 3 tablet (A3 wide) and grip pen.
Figure 6.3: A sample of 20 Chinese characters written by writer A.  

Figure 6.4: A sample of 20 Chinese characters written by writer B.  

Figure 6.5: The complete recognition system with preprocessing. The shaded area implies that pre-processing is optional.  

Figure 6.6: Diagram of the whole preprocessing procedure for the Chinese character '我' (means I or me). Point F is defined as the origin, FG is the x-axis and FH is the y-axis. The y-axis is defined such that moving from F to H implies increasing y values.  

Figure 6.7: Examples of non-normalized (left) and normalized (right) Chinese character: (a) '梦' (means dream), (b) '看' (means see or look), (c) '带' (means bring) and (d) '泪' (means tear).  

Figure 6.8: X-graph (above) and Y-graph (below) of Chinese character '我' (means I or me).  

Figure 6.9: X-graphs (above) and Y-graphs (below) plotted for two similar characters: (a) '白' (means white) and (b) '百' (means hundred).  

Figure 6.10: X-graph (above) and Y-graph (below) plotted for character ‘来’ (means come): (a) Regular character in database, (b) Characters’ '来' written by writer A and (c) Character ‘来’ written by writer B.  

Figure 6.11: Processing time for recognizing characters with varying stroke numbers by using $R_P^2$, MD, CBDD, MQDF, CMF, $R_S^2$, MSSIM and RMSE.  

Figure 6.12: Slant variations in both x-direction and y-direction. Character in black color is the regular writing and the character in red color is slant.  

Figure 7.1: Standard test images.  

Figure 7.2: Samples of compressed images for LENA test image with increasing levels of distortion. JPEG compression with compression factors 1, 50 and 100 (from left to right).  

Figure 7.3: Plots of similarity measures ($R_P^2, R_S^2, R_S^2, MSSIM, RMSE$) against compression factor ($Q_i = 10, 20, ..., 100$) for selected feature vectors: luminance, contrast, entropy, and range. Black colour curve is $R_P^2$ value, red colour curve is its upper limit and blue colour curve is its lower limit.  

Figure 7.4: Biplots ($Q_i, S_i$) for $R_P^2, R_S^2, R_S^2, MSSIM$ and RMSE at different compression factors across 31 test images.  

Figure 7.5: Standard deviations of similarity values at different compression factors across 31 test images.  

Figure 7.6: Length of confidence interval for $R_P^2$ at different compression factors across images.  

Figure 7.7: Example of perfect reference image (left) and imperfect reference
image (right) with Gaussian noise \((\mu = 0, \sigma^2 = 0.001)\). These images are Bikes, Caps, House, Lighthouse2 and Monarch.

Figure 7.8: \((Q_i, S_i)\) plots using perfect reference image (left) and imperfect reference image (right).

Figure 7.9: From left to right: Lena, Baboon, Airplane, Bridge, Boat and Peppers images. From top to bottom: original, decompressed image with factor 74, and decompressed image with factor 50.

Figure 7.10: Generated image and distorted images. J: simple binary image, J1: distorted image with \(n_d = 100\), J2: distorted image with \(n_d = 2000\), J3: distorted image with \(n_d = 10000\).

Figure 7.11: Relationship between the similarity value (mean quality index) and the percentage of distorted area. Note that \(R_f^2 = R_p^2\).

Figure 7.12: Relationship between the mean similarity/quality index and the percentage of distorted area. Note that \(R_p^2 = R_p^2\), URp2 and LRp2 are the upper and lower limits for 95% confidence interval.

Figure 7.13: Relationship between the percentage of distorted area and \(R_f^2\). The images used (from top to bottom) are Lena, Airplane, Boat and Peppers. Note that \(R_f^2 = R_p^2\).

Figure 8.1: Watermark embedding process.

Figure 8.2: Watermarked Couple image and Saiboat image. Host image, 100%, 60% and 0% of transparency (from left to right).

Figure 8.3: Comparing the similarity values of ISMs against the percentage of transparency of watermark. LRp2 and URp2 are lower and upper confidence limits of Rp2.

Figure 8.4: A sequence of video frames.

Figure 8.5: Location of anatomical points and distance with centre point (reference point).